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Rate-dependent fracture toughness of pure polycrystalline ice

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A series of three-point bend tests using single edge notched testpieces of pure polycrystalline ice have been performed at three different temperatures (−20°C, −30°C and −40°C). The displacement rate was varied from 1 mm/min to 100 mm/min, producing the crack tip strain rates from about 10^{-3} to 10^{-1} s^{-1}. The results show that (a) the fracture toughness of pure polycrystalline ice given by the critical stress intensity factor (K_{IC}) is much lower than that measured from the J—integral under identical conditions; (b) from the determination of K_{IC}, the fracture toughness of pure polycrystalline ice decreases with increasing strain rate and there is good power law relationship between them; (c) from the measurement of the J—integral, a different tendency was appeared: when the crack tip strain rate exceeds a critical value of 6×10^{-3} s^{-1}, the fracture toughness is almost constant but when the crack tip strain rate is less than this value, the fracture toughness increases with decreasing crack tip strain rate. Re-examination of the mechanisms of rate-dependent fracture toughness of pure polycrystalline ice shows that the effect of strain rate is related not only to the blunting of crack tips due to plasticity, creep and stress relaxation but also to the nucleation and growth of microcracks in the specimen.

1. Introduction

It is well known that ice exhibits two kinds of behaviour depending on the loading condition: ductile when slowly loaded and brittle when rapidly loaded. In the temperature between −20°C and −40°C, the transition from ductile yielding to brittle fracture occurs at a strain rate of 10^{-2} s^{-1} in compression tests [1–4] and in a strain rate range of 10^{-4}–10^{-3} s^{-1} in tension tests [1]. Ductile behaviour of ice is controlled by the glide and climb of basal dislocations and by dynamic recrystallization; brittle behaviour of ice by the growth and interaction of cracks [4]. On loading, a non-uniform internal stress field exists in polycrystalline ice aggregates due to the elastic and creep anisotropy of single ice crystals. Its average, of course, is always equal to the applied stress, but the peaks can be many times larger [5]. Therefore, even when the applied stress is small, the internal stresses can cause the nucleation of cracks. The wavelength of the internal stress and the crack size is roughly equal to the grain size [5]. At low strain rates, the cracks do not propagate; instead, new cracks form as the load is increased, and the ice exhibits ductile behaviour. At higher rates, the cracks do propagate and th ice behaves in a brittle fashion. The transition results from a competition between stress buildup and stress relaxation near the crack tips [4]. According to fracture mechanics, crack growth occurs when the incremental energy available from the release of stored potential (strain) energy is equal to, or exceeds, that required to create new fracture surfaces. Resistance to crack growth is defined by fracture toughness (R), which is the amount of work required to propagate a crack by unit area [6]. Alternatively, fracture toughness of a brittle material can be given in terms of K_C, the so-called critical stress intensity factor. R and K_C are related by:

$$K_C^2 = E^* R \tag{1}$$

where E^* is given by E for plane stress or $E/(1-\nu^2)$ for plane strain, in which, E is the Young's Modulus and ν Poisson's ratio.

Fracture toughness of pure ice, usually, in terms of the plane strain mode I tensile critical stress intensity factor K_{IC} (or its candidate value in a possibly "invalid" test, K_Q), has been determined by many researchers [7–13]. The reported values of K_{IC} of ice are very scattered, ranging from 25 kPam$^{1/2}$ to 416 kPam$^{1/2}$. The reason

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for this is that fracture of ice is strongly dependent on the ice type (i.e., microstructure), test temperature, loading rate, specimen preparation and specimen geometry and dimensions [10, 11]. The effect of rate on the fracture toughness of pure ice has been considered by many workers but, even so, it is still not well understood. Goodman and Tabor [7] proposed that the process of cracking in ice involve not only the true surface energy of the solid but also creep and deformation at the crack tip. Liu and Miller [8] measured the values of K_{IC} of fresh-water ice at various temperatures between $-1°C$ and $-46°C$ and at six loading rates, and concluded that the fracture toughness of ice is increased by a decrease in loading rate. This phenomenon was explained as a consequence of stress relaxation and creep of material at the crack tip. Urabe and Yoshitake [9] obtained similar experimental results, and were able to fit a log-log curve to their data to give:

$$K_{IC} = (155 \log T + 61)\dot{K}_{I}^{-(0.08 \log T + 0.03)} \quad (2)$$

where T is the temperature (in K) and $\dot{K}$ the loading rate (in kPam$^{1/2}$ s^{-1}). It was noted that this variation of the fracture toughness with rate might be related to the stress-state at or near crack tip. Nixon and Schulson [10] reported that, when the loading rate $\dot{K}$ is equal to or greater than a value of 10 kPam$^{1/2}$ s^{-1}, K_{IC} of ice at $-10°C$ is a constant; if $\dot{K}$ decreases below this value, the fracture toughness of ice increases. Following Urabe and Yoshitake [9], Nixon and Schulson [10] noted that, for a valid measurement of the plane strain fracture toughness, the size of the creep zone at the crack tip must be small in comparison with other specimen dimensions. In a material such as ice in which crack tip deformation is rate dependent, the transition from plane strain to plane stress can be achieved by reducing the loading rate as well as by reducing the thickness. In that case, the specimen becomes less constrained and then the fracture toughness increases. However, following these mechanisms, we found that the theoretical analysis does not agree with the practical measurements. This paper attempts to give a further understanding of the rate effect on the fracture toughness of pure polycrystalline ice.

2. Experimental procedure

As ice is a material very sensitive to temperature, microstructure, etc., variation of the cooling conditions will markedly change its structure and properties. In attmepting to achieve a uniform and fine crystal structure, a special method for preparation of specimens was developed. The pre-frozen blocks of pure water were finely comminuted by the blades of a food mixer blender which achieved an ice fragment diameter of about 0.1–0.2 mm. Then the fragments were made into a poultice by adding a little water and filled into a mould box made of cardboard. The samples were put into a freezer (at about $-20°C$) for over 24 h. Before testing, the cardboard was peeled off and th testpieces were kept in the thermal cabinet for 30–45 min at the testing temperature to equilibrate.

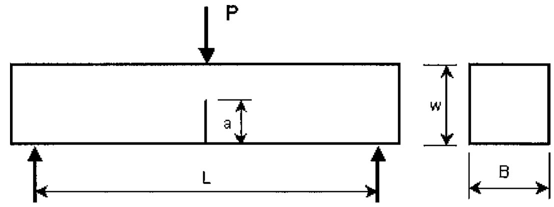

Figure 1 The geometry of the single edge notched testpiece.

Single edge notched testpieces in three-point bending (Fig. 1) were chosen since the configuration can be easily shaped and tested. The beam size was 25 mm × 25 mm × 160 mm. Notches were carefully formed by melting using a metal knife and a plastic mould. In standards [14–16], a range of 0.4 to 0.7 for the relative notch depth (the ratio of pre-crack length a to depth of beam w) is usually recommended. Some researchers suggested that an appropriate crack length should be $0.2 \leq a/w \leq 0.35$ [11, 17]. In the present study, the relative notch depth was about 0.4. Due to the size of the notching tools, the notch width was between 1.5 and 2.0 mm (see Fig. 2). The dependence of the fracture toughness on the crack-tip radius is well known for many materials. Theoretically, a zero root radius of crack tip is needed to satisfy the requirement of linear elastic fracture mechanics (LEFM). For conventional engineering materials, starter cracks are sharpened in fatigue. The extension of the crack is at least $0.05w$ ahead of the notch to eliminate the effects of the geometry of the machined notch. The effects of notch acuity (crack-tip sharpness) on the fracture toughness of ice were investigated by Wei *et al.* [13] using six groups of SENB specimens with different crack (or notch) root radii. The results show that the mean value and standard deviations of the apparent fracture toughness values K_Q of the specimens with blunt notches were significantly higher than those of the specimens with sharp cracks. Consistent K_Q values can be obtained by employing a razor cut either with or without pre-load for fabricating the initial naturally sharp macrocrack in the fracture toughness testing of ice. There were no significant difference between the measured fracture toughness of notched and fatigued pre-cracked specimens. Same observation was also reported by Liu and Miller [8]. In the present study, these methodologies were abandoned because of the great problems in controlling the fatigue crack growth. Even so, a small root radius of crack tip can be produced when the notching process is by *melting* rather than by cutting.

The experiments have been carried out at three different temperatures of -20, -30 and $-40°C$ and

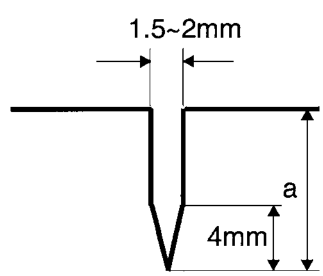

Figure 2 The geometry of the notch.

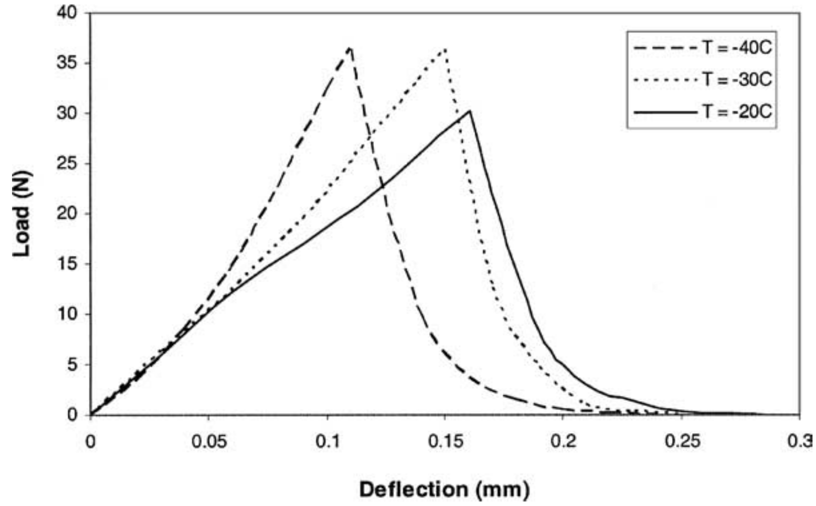

Figure 3 Typical load-displacement curves of pure polycrystalline ice (Crosshead speed: 5 mm/min).

in the range of crosshead speed from 1 mm/min to 100 mm/min, using an Instron Tensile Testing Machine. The temperature was controlled using a thermal cabinet and the coolant was liquid CO_2. The typical load-displacement curves at three temperatures with a crosshead speed of 5mm/min during the tests are shown in Fig. 3. From which, we can see that the fracture of pure polycrystalline ice is quite brittle and the crack growth might be looked as of stable.

There are several ways to obtain the value of fracture toughness. The critical stress intensity factor, K_{IC}, has been widely employed for brittle materials. K_{IC} is determined as a function of the critical applied stress and corresponding pre-crack length. Its expression, for the single edge notched beam in three-point bend tests is given by [18–21]

$$K_{IC} = \frac{3}{2}\frac{P_c L}{B w^{3/2}} Y(a/w) \tag{3}$$

where P_c is the critical applied load, L the span of beam, B the specimen thickness, w the specimen depth, a the pre-crack length and $Y(a/w)$ is the calibration function. In most standards and published works, the tabulated values or closed form expressions for $Y(a/w)$ are accurate only over a limited range of a/w (for examample, $0.4 \le a/w \le 0.7$). The span-to-depth ratio of L/w is often fixed at a value of 4. General expressions for the stress intensity factor for any value of the a/w and L/w larger than 2.5 have been derived by Guinea *et al.* [22]. Writing $\alpha = a/w$ and $\beta = L/w$, the calibration function $Y(a/w)$ is replaced by a general shape function $k_\beta(\alpha)$.

$$k_\beta(\alpha) = \frac{\sqrt{\alpha}}{(1-\alpha)^{3/2}(1+3\alpha)} \times \left\{ p_\infty(\alpha) + \frac{4}{\beta}[p_4(\alpha) - p_\infty(\alpha)] \right\} \tag{4}$$

in which

$$p_4(\alpha) = 1.9 + 0.41\alpha + 0.51\alpha^2 - 0.17\alpha^3 \tag{5}$$

$$p_\infty(\alpha) = 1.99 + 0.83\alpha - 0.31\alpha^2 + 0.14\alpha^3 \tag{6}$$

An alternative approach to characterise fracture resistance is the so-called energy line integral, J, which is used to characterise either elastic or small-scale elastic-plastic behaviour in the process zone. For reversible elasticity (linear or non-linear), J is the energy being made available at the crack tip per unit increase in crack area. In other words, it is a measure of the elastic energy release rate. For compact tension and bend specimens, J can be approximated [23] by

$$J_I = \frac{2U}{B(w-a)} \tag{7}$$

where U is potential energy of the loaded body, B is the specimen thickness, w is the specimen depth and a is crack length. Cotterell and Atkins [24] showed that at the initiation of fracture, the critical value of the J-integral, J_{IC}, is equal to the material fracture toughness, R. Thus

$$R = J_{IC} = \frac{2U_C}{B(w-a)} \tag{8}$$

where U_C is the work done by the applied load before the crack begins.

In previous studies, K_{IC} for ice has been given as a function of loading rate or displacement rate rather than the crack tip strain rate [7–10]. Unfortunately different geometry specimens can have quite different strain rates for the same crosshead speed, and in order to study the kinetics of the mechanisms that contribute to crack resistance under different conditions, it is necessary to have the data in terms of crack tip strain rate. For an elastic SENB specimen during three-point bending, the crack tip strain rate may be written in terms of a semi-empirical expression [25]:

$$\dot{\varepsilon} = \left[2.3357\left(\frac{\sigma_{nc}}{\sigma_y}\right)^4 - 6.335\left(\frac{\sigma_{nc}}{\sigma_y}\right)^3 + 8.5633\left(\frac{\sigma_{nc}}{\sigma_y}\right)^2 + 2.0191\left(\frac{\sigma_{nc}}{\sigma_y}\right) \right] \frac{P}{P_c}\frac{\dot{\delta}}{\delta_c}\varepsilon_y \tag{9}$$

in which,

$$\sigma_{nc} = \frac{3}{2}\frac{P_c L}{B(w-a)^2} \tag{10}$$

where $\dot{\varepsilon}$ is the crack tip strain rate, ε_y is uniaxial yield strain, σ_y is uniaxial yield stress, σ_{nc} and δ_c are the nominal bending stress and the displacement at the critical load P_c, P is the applied load, L the beam span, B the specimen thickness, w the specimen depth, a the pre-crack length and $\dot{\delta}$ is the displacement rate. In the present study, the critical applied load P_c in Equations 3, 9 and 10 is the maximum applied load.

3. Experimental results and discussions

The critical stress intensity factors of pure polycrystalline ice have been calculated using Equations 3–6 and their variations at different temperature with crack tip strain rate are shown in Fig. 4. The values lie between 22 to 169 $\mathrm{kPam}^{1/2}$, and the variation with rate is in good agreement with many previous investigations [7–12]. The critical stress intensity factors of pure polycrystalline ice at all three temperatures decrease with increasing crack tip strain rate and there are good power law relationships between them. That is, for a given temperature,

$$K_{IC} \propto \dot{\varepsilon}^{-n} \tag{11}$$

where K_{IC} is the critical stress intensity factor, $\dot{\varepsilon}$ is crack tip strain rate and n is a positive constant. As mentioned before, this was explained by Liu and Miller [8] in terms of the stress relaxation and creep deformation at the crack tip. They argued that when the same applied K is reached by specimens loaded at various $\dot{K}$, the crack tip stress of the specimen at the lower $\dot{K}$ is less because of stress relaxation. The applied K (at a lower $\dot{K}$) has to reach a higher level in order to produce fracture. In terms of energy, one may say that the stress relaxation reduces the available potential energy for crack extension. If it is believed that the critical stress intensity factor of pure polycrystalline ice is mainly dominated by the stress relaxation or creep of material at crack tip, the relationship given by Equation 11 ought to be obtainable in the following way.

For pure polycrystalline ice, the steady-state creep can be expressed as a power law [5, 26, 27]:

$$\dot{\varepsilon} = A\sigma^{m} \tag{12}$$

where $\dot{\varepsilon}$ is creep strain rate, σ is stress, m is a positive index and A is a temperature-related constant. The general expression for stress intensity factor is:

$$K_{IC} = \sigma_c\sqrt{\pi a}Y(a/w) \tag{13}$$

where σ_c is the critical stress applied normal to the central plane of the Mode I crack. Substituting for σ from Equations 12 to 13, we have:

$$K_{IC} \propto \dot{\varepsilon}^{\frac{1}{m}} \tag{14}$$

Clearly, Equations 11 and 14 do not agree each other. Equation 14 indicates that the stress intensity factor should increase with increasing strain rate (m is positive), while Equation 11 shows that the stress intensity factor decreases. The stress relaxation and creep effects on fracture toughness for other materials (for example, metals and polymers) have been discussed by Atkins and Mai [6]. They considered the variation of Young's modulus and flow stress with strain rate and obtained:

$$K_C \propto t^{-(n_0+m_0)/2} \tag{15}$$

where t is time and n_0 and m_0 are the positive constants. It can be seen that Equation 15 has an identical form to Equation 14 since the strain rate is the reciprocal of time. This relationship, that the stress intensity factor increases with increasing strain rate, is supported by the results of Birch and Williams [28] on polymers. Since the experimental results for ice contradict the above theoretical analysis, the mechanism of stress relaxation and creep at the crack tip may be not able of itself to

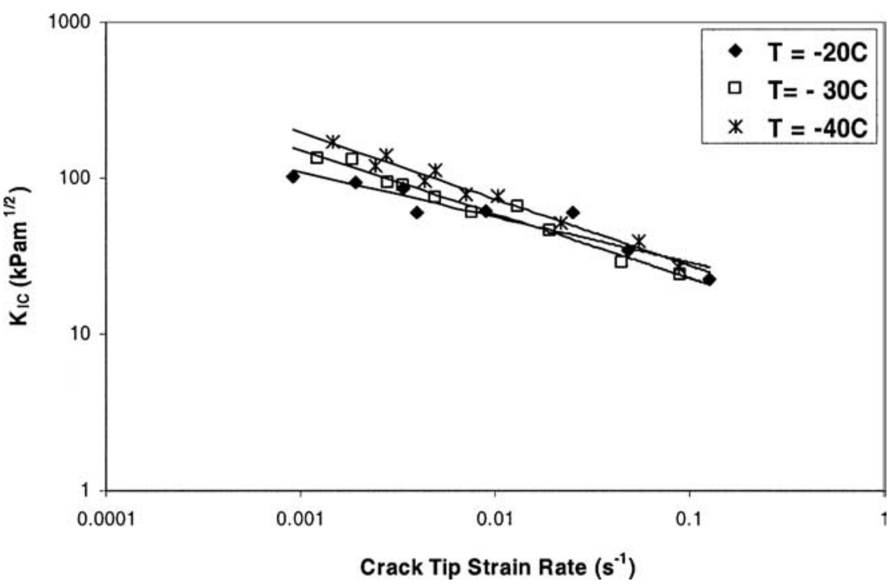

Figure 4 Variation of K_{IC} as a function of crack tip strain rate.

explain the cause of the rate-dependent fracture toughness of pure polycrystalline ice.

For a plane strain condition to exist in a material exhibiting rate independent plasticity, it is well accepted that the crack tip plastic zone must be less than 1/50th of the smallest critical dimension of the specimen. Nixon and Schulson [10] assumed that this limitation can be still valid for the creep zone size at crack tips in a rate-dependent material such as pure ice. On loading, the crack tip stress and strain behaviour changes with time, as a result of continuous evolution of the creep zone. For steady-state creep, the power law Equation 12 can be still used to describe the relationship between the stress and strain rate near the crack tip. The size of creep zone can be determined by using the HHR-creep strain and the remote elastic singular strain field [29]:

$$r_{\mathrm{cr}} = \frac{(K_{\mathrm{I}})^2}{2\pi} \left\{ \frac{(m+1) I_{\mathrm{m}} E A t}{2\pi(1-\nu^2)} \right\}^{\frac{2}{m-1}} F_{\mathrm{cr}}(\theta) \qquad (16)$$

where r_{cr} is the creep zone size, K_{I} is the stress intensity factor, E is Young's Modulus, ν is Poisson's ratio, A and m are creep strain rate constants, t is loading time, I_{m} is a dimensionless parameter, $F_{\mathrm{c}}(\theta)$ is the angular function describing the variation of the crack tip zone radius. Nixon and Schulson [10] noted that as strain rate (or loading rate) decreases, time t and therefore the creep zone size r_{cr} increases. Eventually, r_{cr} will be large enough to violate the plane strain condition. For pure polycrystalline ice, the values of m, A, E, ν and I_{m} in Equation 16 may be found in the literature [25, 30, 31] and are given in Table I. Whether for plane strain or for plane stress, the value of $F_{\mathrm{c}}(\theta)$ is usually less than unity [31]. Taking $F_{\mathrm{c}}(\theta) = 1$ and using the data for K_{IC} and t obtained in the present study, the creep zone sizes of specimens at different temperatures and at different strain rates have been calculated and the results are shown in Fig. 5. As expected, the creep zone size decreases with increasing strain rate due to the reduction of the time scale. But, even at the lowest crack tip strain rate used, the creep zone sizes (ca. 0.015–0.034 mm) are still much less than the dimension limitation (the smallest dimension of the specimen is the pre-notch length which is about 10 mm and the corresponding limiting creep zone size should be 0.2 mm). This indicates that the critical stress intensity factor varying with crack tip strain rate may be related to the variation of the creep zone size but, at least in the present measurement, not to the transition of stress state (i.e., the transition from plane strain to plane stress) of specimen.

TABLE I Values of the parameters in Equation 16

	m	$B[\mathrm{s}^{-1}(\mathrm{MPa})^{-\mathrm{m}}]$	E(GPa)	ν	I_{m}
$T = -20°\mathrm{C}$	4.1	2.2×10^{-9}	9.7	0.33	5.20
$T = -30°\mathrm{C}$	4.3	9.3×10^{-10}	9.9	0.33	5.15
$T = -40°\mathrm{C}$	5.3	4.7×10^{-11}	10.1	0.33	5.00
Reference	[25]	[25]	[30]	[30]	[31]

In previous studies [7, 8], the value of R of pure polycrystalline ice was usually calculated by means of its critical stress intensity factor, K_{IC}, using Equation 1. If both Young's Modulus and Poisson's ratio of pure polycrystalline ice are constants, variations in R with crack tip strain rate are similar to the critical stress intensity factor (see Figs 4 and 6) and, of course, the exponent is equal to twice that in Equation 11 since $K_{\mathrm{IC}}^2 = E * R$. Taking the values of E and ν from Table I, the calculated R lie between 0.06 J/m^2 at the highest strain rate and 3.2 J/m^2 at the lowest strain rate. For a truly brittle solid, it is well known that the fracture toughness should be equal to twice surface energy. The surface energy of pure polycrystalline ice is 0.065 J/m^2 when the fracture is transgranular and 0.11 J/m^2 when the fracture is intergranular [32]. The ideal value of fracture toughness for pure polycrystalline ice is thus from 0.13 to 0.22 J/m^2. The higher value determined in experiments was explained in terms of the contribution of the plastic and creep deformation at the crack tip [7, 8], while the lower values were associated with the existence of a thin liquid film that covers the individual ice crystals [8].

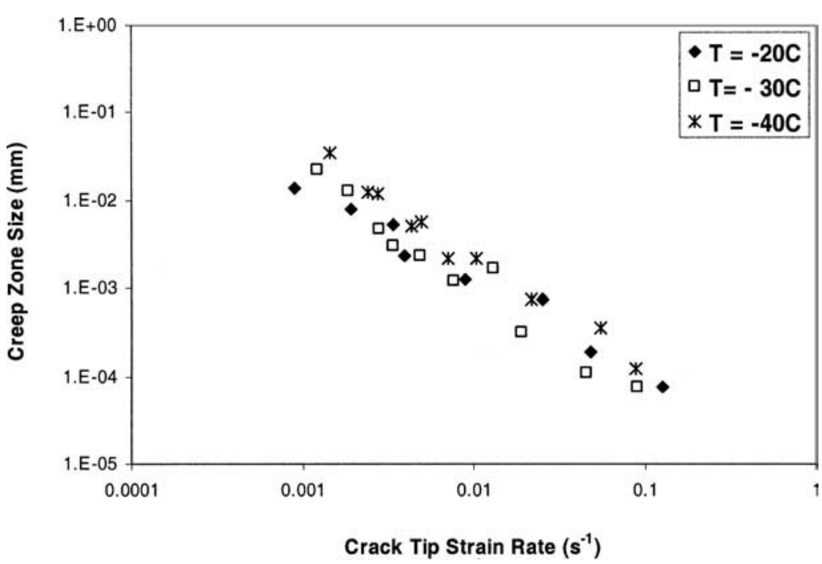

Figure 5 Variation of creep zone size as a function of crack tip strain rate.

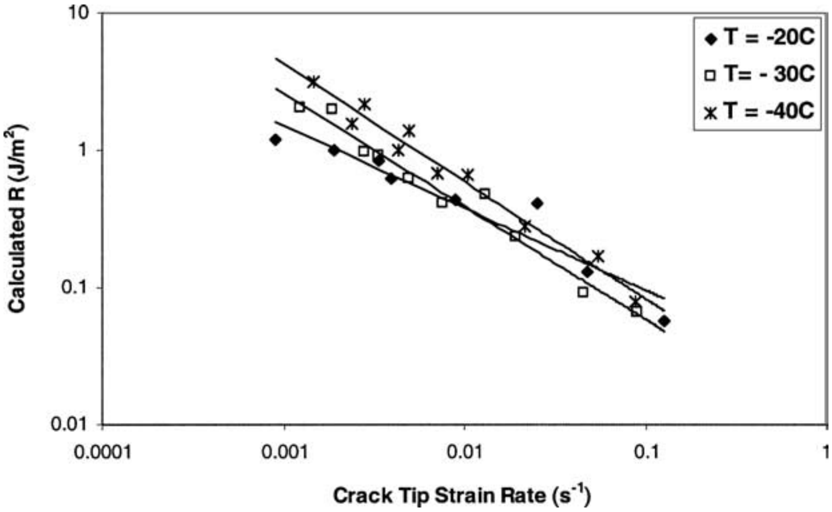

Figure 6 Variation of calculated R as a function of crack tip strain rate.

In terms of Equation 8, the apparent fracture toughness (actual fracture energy) of pure polycrystalline ice has been obtained from the same experiments used to calculate the critical stress intensity factor. Its variation with critical crack tip strain rate at three temperatures is shown in Fig. 7. At low crack tip strain rates, the fracture toughness is greater than that at higher strain rates. With increasing rates, the fracture toughness decreases until it reaches a plateau having almost constant R independent of the crack tip strain rate. The minimum fracture toughness of pure polycrystalline ice is about 8.5 J/m^2, occurring first at a crack tip strain rate of about 6×10^{-3} s^{-1} for all three temperatures. In comparison with Fig. 6, it is obvious that there are the greater differences between the actual value and the calculated value for the fracture toughness of pure polycrystalline ice, whatever on the magnitude or tendency. The actual work done is much greater than the calculated fracture work and the ideal value of the fracture toughness (i.e., twice surface energy). This indicates that, as well as the stress relaxation and creep at the crack tip, there must be other events which consume energy during the fracture process and that these events are rate-dependent.

As mentioned before, on loading, non-uniform microstructural stresses exist in polycrystalline ice aggregates, the peaks of which are many times greater than the mean applied stress. The internal stresses grow with increasing strain, either until a steady state is reached or until one or more cracks form, with a size equal to the wavelengths of the internal stress [5]. The critical stress for crack nucleation in ice has been given by Gold [33]:

$$\tau_e = \left[\frac{\pi\gamma E}{4(1-\nu^2)d}\right]^{\frac{1}{2}} \tag{17}$$

where τ_e is the effective shear stress, γ is the surface energy, E is Young's Modulus, ν is Poisson's ratio and d is the grain diameter. Gold [33] calculated a value of 0.48 MPa for τ_e when $d = 3$ mm; in experiments he observed that crack formation takes place when the stress exceeds about 0.6 MPa in compression in the temperature range of -5 to -31°C. For stresses between 0.6 to 1.2 MPa most cracking activity occurs. If the stress is greater than about 1.2 MPa, deterioration of the structure due to cracking causes the primary stage of creep to be transformed directly to the tertiary stage. The values

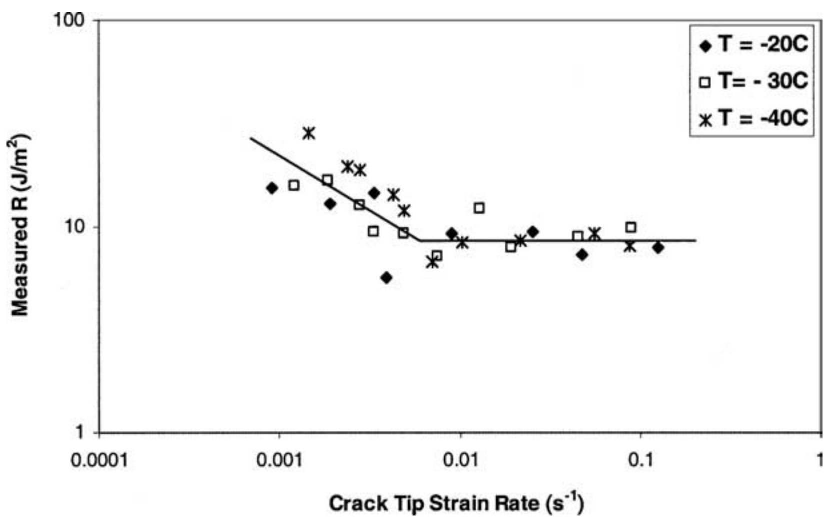

Figure 7 Variation of measured R as a function of crack tip strain rate.

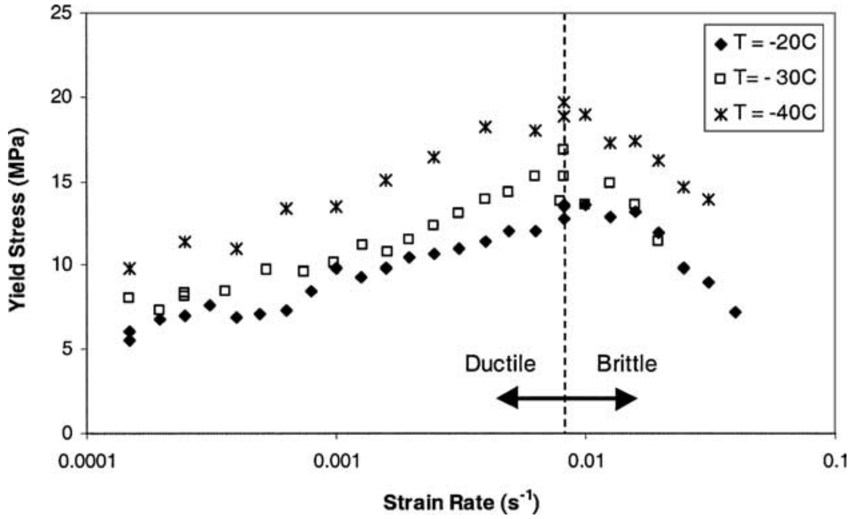

Figure 8 Variation of yield stress as a function of strain rate.

of stress at which cracks first appear in specimens of polycrystalline ice during deformation have been reported by several investigators. Hawkers and Mellor [1] reported the stress in compression to be 2–4 MPa at $-7°C$, while Duval *et al.* [5] and Schulson [4] showed that at a temperature of $-10°C$, a compressive stress of 1.5 MPa would cause the nucleation of cracks. At the very low temperature of $-110°C$, the first cracks appeared at a tension stress of about 0.3 MPa [33]. Clearly, consideration of pure polycrystalline ice subjected to rather low stresses should certainly include microcracks, the number of which increases with increasing stress and time [33–35]. At lower strain rates, on the one hand, the cracks continue to nucleate as the stress increases; on the other, the creep and stress relaxation at crack tips lead to crack blunting. At higher strain rates, the creep and stress relaxation at the crack tip become less significant and the stresses build up very quickly. As soon as the stress intensity factor reaches its critical value, microcracks begin to grow and interact, and brittle failure ensues.

The formation of microcracks at low stresses is an important feature for the fracture of pure polycrystalline ice. On the one hand, the nucleation and growth of a great number of microcracks may greatly increase the work consumption and reduce the capacity of specimen to bear load; on the other hand, the specimen at lower strain rates has a higher fracture resistance than that at a higher strain rate due to the sensitivity of the crack to stresses. In order to determine the yield stress and Young's modulus of pure polycrystalline ice, Xu [25] has carried out a series of indentation and simple three-point bend (with unnotched beam) experiments. The results are shown in Figs 8 and 9. It can be seen that variation of both yield stress and apparent Young's modulus of pure polycrystalline ice with strain rate are similar. The yield stress and apparent Young's modulus increase initially with increasing strain rate before reaching a peak. At a critical strain rate, the ductile-brittle transition occurs; the yield stress and the apparent Young's modulus reach their maximum. Afterwards, the yield stress and the apparent Young's modulus decrease with increasing strain rate. The transition strain rates are about $8 \times 10^{-3}\ s^{-1}$ during indentation tests and about $8 \times 10^{-4}\ s^{-1}$ during three-point bend tests. These results confirm that the ductile-brittle

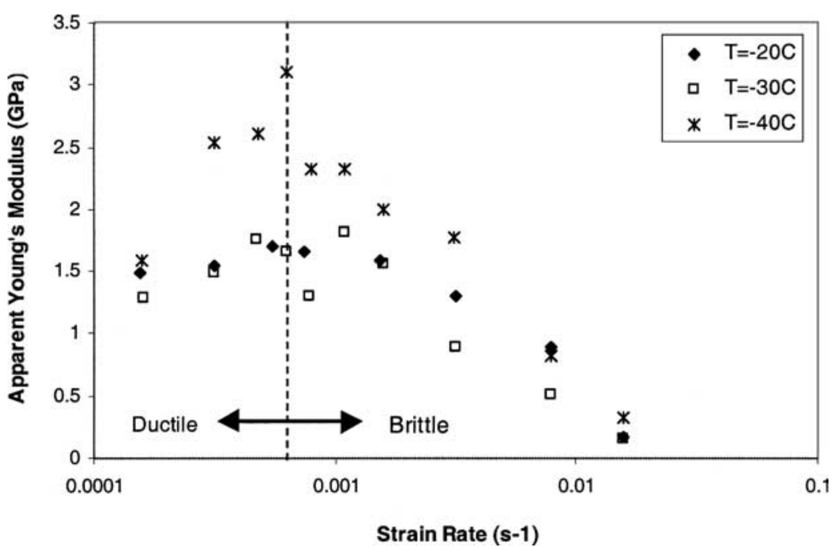

Figure 9 Variation of apparent Young's modulus as a function of crack tip strain rate.

transition of pure polycrystalline ice occurs at different strain rates under compression and tension. The crack tip strain rates produced in three-point bending with a single edge notched beam used in the present study, even at the smallest crosshead speed, are greater than the ductile-brittle transition strain rate in tension (see Figs 4–7). At these higher strain rates, creep and stress relaxation at the main (pre-notch) crack tip does not play an important role in the fracture of pure polycrystalline ice. Comparing Figs 7 and 8, it appears that the transitions occur at the same strain rate level for both indentation tests and fracture tests. This may be understood in terms of the ductile-brittle transition during experiments being dependent on whatever the stress is tensile or compressive. For the un-notched beam under three-point bend (Fig. 10a), the tension edge usually dominates the deformation and fracture process so that the ductile-brittle transition occurs at low strain rate. The nucleation and propagation of micrcracks on the both surfaces reduces the stiffness of the beam. Therefore, the Young's modulus measured by this method is always lower than that from high frequency measurements [27, 31]. For the notched beam (Fig. 10b), only the notch tip lies in the tension stress state. The transition from ductile yielding to brittle fracture is therefore dominated by the compression edge. However, the nucleation and growth of microcracks is significant in setting the level of crack resistance. Therefore, fracture work of pure polycrystalline ice must include three main components: (1) the work to create the apparent new surface (along the direction of the pre-notch); (2) the work used to nucleate and grow the microcracks; and (3) the work dispersed at microcrack tips due to plasticity, creep and stress relaxation. How each part contributes to the total work done under different conditions is yet to be resolved. More investigations are required in this area because it may be very important for ice engineering and other applications. By means of the results in Figs 4 and 7, a mechanism for the effect of rate on the fracture toughness of pure polycrystalline ice may be described as follows.

As soon as a load is applied to pure polycrystalline ice specimens a great number of microcracks are immediately nucleated. The number of microcracks increases with increasing stress, strain and time, and their size increases at increasing strain rate. Both the increases in the number and size of microcracks reduce the area of load-bearing cross-section and hence decrease the apparent fracture load and the critical stress intensity factor. However, a large number of new microcracks means the creation of a large surface area, which will greatly increase the total work done. At strain rates less than the ductile-brittle strain rate, say $6 \times 10^{-3}\ s^{-1}$, microcrack tips would be blunted by plasticity, creep and stress relaxation. Crack tip blunting has two different consequences: decreasing the sensitivity of crack propagation on stress, and increasing the work consumption. With decrease of strain rate, the time scale increases, therefore the number of microcracks increases and the effect of crack tip blunting becomes more significant. As a result, the apparent fracture toughness increases. With increase of strain rate, the size of microcracks increases and the effect of the crack tip blunting decreases, whence both the critical stress intensity factor and the apparent fracture toughness decreases. When the crack tip strain rate exceeds a critical value, i.e., $6 \times 10^{-3}\ s^{-1}$, the nucleation of microcracks becomes limited and the crack tips have no time to creep or relax. Hence, the apparent fracture toughness stays at its minimum value. The continuous decrease of the critical stress intensity factor with crack tip strain rate is related to the incessant extension of the main crack and the increase in size of microcracks at the higher strain rates.

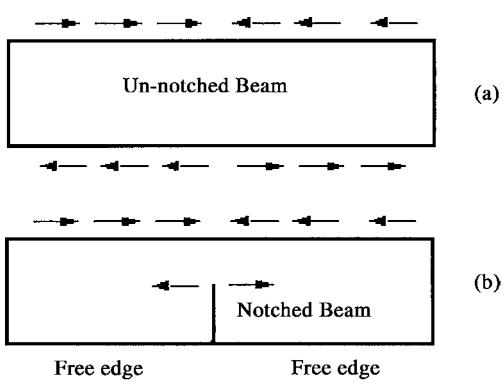

Figure 10 Different stress state for un-notched and notched beams.

4. Conclusions

The effect of crack tip strain rate on the fracture toughness, R, of pure polycrystalline ice is very complex, concerning not only the blunting mechanism of crack tips due to plasticity, creep and stress relaxation but also the nucleation and growth of microcracks in the specimen. (1) When the crack tip strain rate exceeds a critical value of $6 \times 10^{-3}\ s^{-1}$, the fracture toughness is almost a constant and independent of the temperature and crack tip strain rate. This is because the time scale is very small so that the nucleation of microcracks has reached a limiting value and no creep and relaxation occurs at the crack tips. (2) When the crack tip strain rate is less than the critical value, the fracture toughness decreases with increasing crack tip strain rate due to the increase in the number of microcracks and plastic and creep deformation. (3) The decrease of the critical stress intensity factor, K_{IC}, with increasing crack tip strain rate can be considered as a result of the incessant extension of all cracks (prenotch and microcracks).

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